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DISCUSSION ABOUT THE ANALYTICAL CALCULATION OF THE MAGNETIC FIELD CREATED BY PERMANENT MAGNETS

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Abstract—This paper presents an improvement of the calculation of the magnetic field components created by ring permanent magnets. The three-dimensional approach taken is based on the Coulombian Model. Moreover, the magnetic field components are calculated without using the vector potential or the scalar potential. It is noted that all the expressions given in this paper take into account the magnetic pole volume density for ring permanent magnets radially magnetized. We show that this volume density must be taken into account for calculating precisely the magnetic field components in the near-field or the far-field. Then, this paper presents the component switch theorem that can be used between infinite parallelepiped magnets whose cross-section is a square. This theorem implies that the magnetic field components created by an infinite parallelepiped magnet can be deducted from the ones created by the same parallelepiped magnet with a perpendicular magnetization. Then, we discuss the validity of this theorem for axisymmetric problems (ring permanent magnets). Indeed, axisymmetric problems dealing with ring permanent magnets are often treated with a 2D approach. The results presented in this paper clearly show that the two-dimensional studies dealing with the optimization of ring permanent magnet dimensions cannot be treated with the same precisions as 3D studies.

1. INTRODUCTION

This paper continues the papers written by Babic and Akyel [1] and Ravaud et al. [2]. Ring permanent magnets axially or radially magnetized are commonly used for creating magnetic fields in magnetic bearings [3, 4], in flux confining devices [5–9], in sensors [10–12], in electrical machines [13–16] and in loudspeakers [17–19]. The calculation of the magnetic field created by such structures can be done by using numerical methods or analytical methods. The numerical methods are often based on a finite element method but the evaluation of the magnetic field components with such methods has a very high computational cost. Analytical methods are either based on the Coulombian model [20–23], or the Amperian model [24–33], in a 2D or 3D approach. Authors have often used 2D-analytical methods for optimizing ring permanent magnet dimensions because their expressions are fully analytical. However, these expressions are not valid in two cases: for small ring permanent magnets and when the magnetic field is calculated far from the magnets. Consequently, 3D-analytical methods are required.

As emphasized at the beginning of the introduction, such methods have already been presented, but this paper improves once more the way of calculating the magnetic field created by a ring permanent magnet radially magnetized. Indeed, the magnetic pole volume density is taken into account and we show that this magnetic charge contribution is necessary to calculate the magnetic field components in the near-field and the far-field. Then, we discuss the validity of the component switch theorem for ring permanent magnets and we show that this theorem cannot be used in three-dimensions whereas it can be used in two dimensions between two infinite parallelepiped magnets. The main reason lies in the fact that the magnetization is uniform for a ring permanent magnet axially magnetized whereas it is not for a ring permanent magnet radially magnetized. Such results are important for modeling the magnetic field created by cylindrical structures. It is noted that the 2D approximation consists in representing a ring permanent magnet by an infinite parallelepiped as shown in Fig. 1. With this approximation, the two magnetic components $H_r(r,z)$ and $H_z(r,z)$ created by an infinite parallelepiped are assumed to be the same as the ones created by a ring permanent magnet radially magnetized (see Fig. 2). In fact, this approximation is false for two reasons. The first reason has been studied in a previous paper [2]: the magnitude of the magnetic field created by a ring permanent magnet is under-estimated with the 2D approximation. In addition, there is another physical reason which can be demonstrated mathematically. With the 2D approximation, the magnetic field components $H_r(r,z)$ and $H_z(r,z)$ verify the component switch theorem. This theorem implies that the magnetic field components created by an infinite parallelepiped magnet can be deducted from the ones created by the same parallelepiped magnet with a perpendicular magnetization. However, the theorem cannot be used for modelling the magnetic field

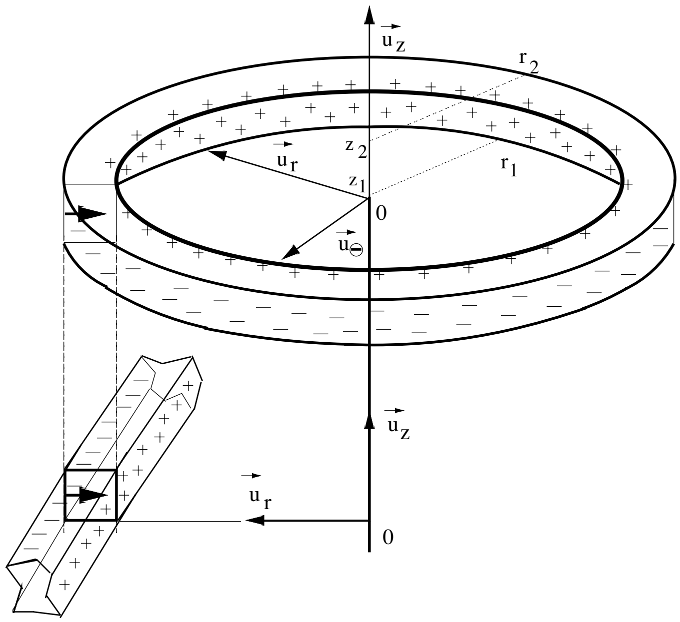


Figure 1. Approximation of a ring permanent magnet with an infinite parallelepiped (the 2D approximation).

components created by ring permanent magnets radially magnetized.

The first section presents the component switch theorem for the case of two infinite parallelepiped magnets. Then, the second section presents the expressions of the magnetic field components created by ring permanent magnets radially magnetized. Then, this section discusses the validity of the component switch theorem for ring permanent magnets axially and radially magnetized.

2. COMPONENT SWITCH THEOREM IN TWO DIMENSIONS

This section presents the component switch theorem that can be used for calculating the magnetic components created by infinite magnetized parallelepipeds.

2.1. The 2D Approximation

Let us consider the two geometries shown in Fig. 2. The upper geometry (Fig. 2(a)) corresponds to an infinite parallelepiped whose

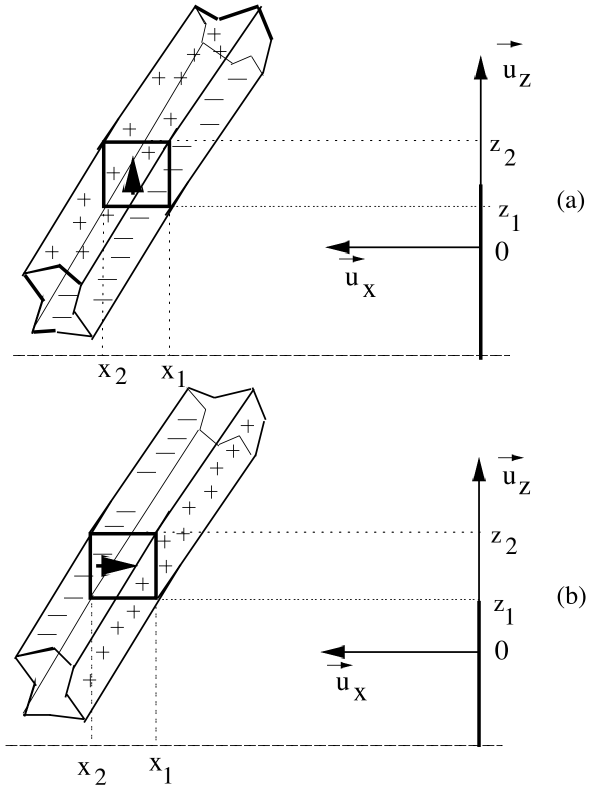


Figure 2. Representation of two infinite parallelepiped magnets whose magnetization undergoes a rotation of 90 degrees between the upper and lower configurations.

polarization is colinear with the z direction. The lower geometry (Fig. 2(b)) corresponds to an infinite parallelepiped whose polarization is colinear with the x direction. As the two configurations studied are infinite along one direction (y direction), the magnetic field they create does not depend on y. Moreover, only two magnetic field components $H_x(x, z)$ and $H_z(x, z)$ exist. These components can be determined analytically by using the coulombian model. By denoting $H_x^{(1)}(x, z)$ and $H_z^{(1)}(x, z)$, the magnetic components created by the configuration shown in Fig. 2(a) and $H_x^{(2)}(x, z)$ and $H_z^{(2)}(x, z)$, the magnetic components created by the configuration shown in Fig. 2(b), we have:

$$H_x^{(1)}(x,z)=\tilde{J}\left(\int\int_{S_{1+}}\frac{\mathbf{PM_{i,i,2}}}{|\mathbf{PM_{i,i,2}}|^3}dS_{1+}-\int\int_{S_{1-}}\frac{\mathbf{PM_{i,i,1}}}{|\mathbf{PM_{i,i,1}}|^3}dS_{1-}\right)\cdot\vec{u}_x \quad (1)$$

$$H_z^{(1)}(x,z)=\tilde{J}\left(\int\int_{S_{1+}}\frac{\mathbf{PM_{i,i,2}}}{|\mathbf{PM_{i,i,2}}|^3}dS_{1+}-\int\int_{S_{1-}}\frac{\mathbf{PM_{i,i,1}}}{|\mathbf{PM_{i,i,1}}|^3}dS_{1-}\right)\cdot\vec{u}_z \quad (2)$$

$$H_x^{(2)}(x,z)=\tilde{J}\left(\int\int_{S_{2+}}\frac{\mathbf{PM_{1,i,i}}}{|\mathbf{PM_{1,i,i}}|^3}dS_{2+}-\int\int_{S_{2-}}\frac{\mathbf{PM_{2,i,i}}}{|\mathbf{PM_{2,i,i}}|^3}dS_{2-}\right)\cdot\vec{u}_x \quad (3)$$

$$H_z^{(2)}(x,z)=\tilde{J}\left(\int\int_{S_{2+}}\frac{\mathbf{PM_{1,i,i}}}{|\mathbf{PM_{1,i,i}}|^3}dS_{2+}-\int\int_{S_{2-}}\frac{\mathbf{PM_{2,i,i}}}{|\mathbf{PM_{2,i,i}}|^3}dS_{2-}\right)\cdot\vec{u}_z \quad (4)$$

where $\tilde{J}=\frac{J}{4\pi\mu_0}$, $\mathbf{PM}_{\alpha,\beta,\gamma}=(x-x_\alpha)\vec{u}_x+(y-y_\beta)\vec{u}_y+(z-z_\gamma)\vec{u}_z$ and $dS_{1+}=dS_{1-}=dx_idy_i$ and $dS_{2+}=dS_{2-}=dy_idz_i$.

The analytical integrations of (1), (2), (3) and (4) give the following expressions:

$$H_x^{(1)}(x,z)=\log\left[\frac{(x-x_2)^2+(z-z_2)^2}{(x-x_1)^2+(z-z_2)^2}\right]+\log\left[\frac{(x-x_1)^2+(z-z_1)^2}{(x-x_2)^2+(z-z_1)^2}\right] \quad (5)$$

$$\begin{aligned}H_z^{(1)}(x,z)=&\arctan\left[\frac{x-x_1}{z-z_2}\right]-\arctan\left[\frac{x-x_2}{z-z_2}\right]-\arctan\left[\frac{x-x_1}{-z+z_2}\right]\\&+\arctan\left[\frac{x-x_2}{-z+z_2}\right]-\arctan\left[\frac{x-x_1}{z-z_1}\right]+\arctan\left[\frac{x-x_2}{z-z_1}\right]\\&-\arctan\left[\frac{x-x_1}{-z+z_1}\right]+\arctan\left[\frac{x-x_2}{-z+z_1}\right] \quad (6)\end{aligned}$$

$$\begin{aligned}H_x^{(2)}(x,z)=&\arctan\left[\frac{x-x_1}{z-z_2}\right]-\arctan\left[\frac{x-x_2}{z-z_2}\right]-\arctan\left[\frac{x-x_1}{-z+z_2}\right]\\&+\arctan\left[\frac{x-x_2}{-z+z_2}\right]-\arctan\left[\frac{x-x_1}{z-z_1}\right]+\arctan\left[\frac{x-x_2}{z-z_1}\right]\\&-\arctan\left[\frac{x-x_1}{-z+z_1}\right]+\arctan\left[\frac{x-x_2}{-z+z_1}\right] \quad (7)\end{aligned}$$

$$H_z^{(2)}(x,z)=-\log\left[\frac{(x-x_2)^2+(z-z_2)^2}{(x-x_1)^2+(z-z_2)^2}\right]-\log\left[\frac{(x-x_1)^2+(z-z_1)^2}{(x-x_2)^2+(z-z_1)^2}\right] \quad (8)$$

Thus, we deduct the following expressions that constitute the component switch theorem.

$$H_z^{(1)}(x,z)=H_x^{(2)}(x,z) \quad (9)$$

and

$$H_x^{(1)}(x,z)=-H_z^{(2)}(x,z) \quad (10)$$

The relations (9) and (10) are valid only if the cross-section of the bar-shaped magnet is a square. The calculations of such expressions are well-known in the literature. As many authors have modelled ring permanent magnets with parallelepiped magnets (2D approximation), it can be interesting to know if the component switch theorem can be used for ring permanent magnets radially and axially magnetized. For this purpose, we propose in the next section to use the coulombian model for calculating the magnetic field expressions created by a ring permanent magnet radially magnetized. It is noted that this calculation has been improved because the magnetic pole volume density is taken into account in this paper and the expressions have been simplified.

3. THREE-DIMENSIONAL EXPRESSIONS OF THE MAGNETIC FIELD COMPONENTS CREATED BY RING PERMANENT MAGNETS

3.1. Notation and Geometry

The geometry considered and the related parameters appear in Fig. 3. The ring inner radius is r_1 and the ring outer one is r_2. Its height is $h = z_2 - z_1$. In addition, the axis z is an axis of symmetry. Calculations are obtained by using the Coulombian model. Consequently, the ring permanent magnet is represented by two curved planes that correspond

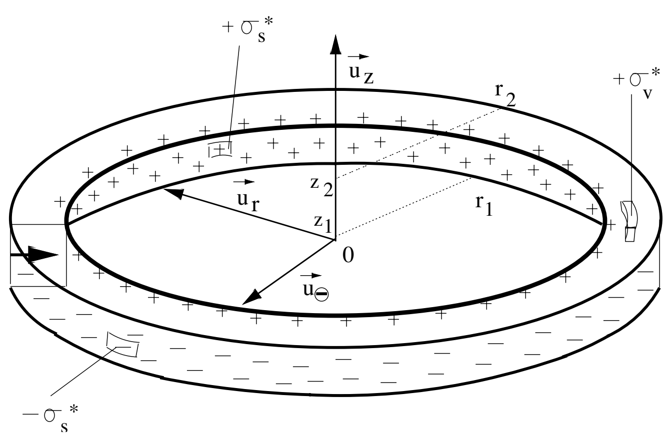


Figure 3. Representation of the configuration studied; the inner radius is r_1, the outer radius is r_2, the height is $z_2 - z_1$, $\sigma_s^* = \vec{J} \cdot \vec{n} = 1\,\mathrm{T}$, $\sigma_v^* = -\nabla \vec{J} = \frac{J}{r}$.

to the inner and outer faces of the ring and a charged ring with a magnetic pole volume density $+\sigma_v^*$. The inner face is charged with a surface magnetic pole density $+\sigma_s^*$ and the outer one is charged with the opposite surface magnetic density $-\sigma_s^*$. By denoting r_0, a point that belongs to the ring permanent magnet, the magnetic field $\vec{H}(r,z)$ created by the ring permanent magnet at any point of the space is expressed as follows:

$$\begin{aligned}\vec{H}(r,z) \;=\; & \int_{(V)} \frac{\sigma_v^*(\vec{r_0})(\vec{r}-\vec{r_0})}{4\pi\mu_0|\vec{r}-\vec{r_0}|^3} d^3\vec{r_0} + \int_{(S_{in})} \frac{\sigma_s^*(\vec{r_0})(\vec{r}-\vec{r_0})}{4\pi\mu_0|\vec{r}-\vec{r_0}|^3} d^2\vec{r_0} \\ & - \int_{(S_{out})} \frac{\sigma_s^*(\vec{r_0})(\vec{r}-\vec{r_0})}{4\pi\mu_0|\vec{r}-\vec{r_0}|^3} d^2\vec{r_0} \end{aligned} \tag{11}$$

with

$$\frac{(\vec{r}-\vec{r_0})}{|\vec{r}-\vec{r_0}|^3} = \frac{(r-r_i\cos(\theta))\vec{u_r} - r_i\sin(\theta)\vec{u}_\theta + (z-z_s)\vec{u}_z}{(r^2+r_i^2-2rr_i\cos(\theta)+(z-z_s)^2)^{\frac{3}{2}}} \tag{12}$$

where $i=1$ for $d\vec{r_0}^2 = dS_{in} = r_1 d\theta_s dz_s$, $i=2$ for $d\vec{r_0}^2 = dS_{out} = r_2 d\theta_s dz_s$, $i=s$ for $d\vec{r}_0^3 = r_s dr_s d\theta_s dz_s$.

3.2. Components Along the Three Directions $\vec{u}_r$, $\vec{u}_\theta$, $\vec{u}_z$

The calculation of (11) leads to the magnetic field components along the three defined axes: $H_r(r,z)$, $H_\theta(r,z)$ and $H_z(r,z)$. It is noted that the azimuthal component $H_\theta(r,z)$ equals 0 because of the cylindrical symmetry.

3.3. Radial Component $H_r(r,z)$

The radial component $H_r(r,z)$ is given by

$$H_r(r,z) = \sum_{i=1}^{2}\sum_{j=1}^{2}(-1)^{(i+j)}\left(S_{i,j}^r + V_{i,j}^r + \mathbf{N}_{i,j}\right) \tag{13}$$

where

$$S_{i,j}^r = \alpha_{i,j}^{(0)}\left(\alpha_i^{(1)}\mathbf{F}^*\left[\alpha_{i,j}^{(2)},\alpha_i^{(3)}\right] + \alpha_i^{(4)}\Pi^*\left[\alpha_{i,j}^{(2)},\alpha_i^{(5)},\alpha_i^{(6)}\right]\right) \tag{14}$$

$$\begin{aligned} V_{i,j}^r \;=\; & f(z-z_j, r^2+r_i^2+(z-z_j)^2, 2rr_i, -1) \\ & - f(z-z_j, r^2+r_i^2+(z-z_j)^2, 2rr_i, 1) \end{aligned} \tag{15}$$

with

$$f(a,b,c,u) = -\eta\left(\beta^{(1)} + \beta^{(2)}\right) \tag{16}$$

$$\begin{aligned}\eta \;=\; & \beta^{(3)}\left[(b-c)\mathbf{E}^*\left[\beta^{(4)},\beta^{(5)}\right] + c\mathbf{F}^*\left[\beta^{(4)},\beta^{(5)}\right]\right] \\ & +\beta^{(6)}\left[(b-a^2)\mathbf{F}^*\left[\beta^{(7)},\beta^{(8)}\right] + (b-a^2+c)\Pi^*\left[\beta^{(9)},\beta^{(7)},\beta^{(8)}\right]\right] \\ & -\beta^{(10)} - \beta^{(11)}\end{aligned} \tag{17}$$

Table 1. Parameters used for calculating the surface contribution of the radial component $H_r(r,\theta,z)$.

Parameters	
$\alpha_{i,j}^{(0)}$	$\frac{J\sqrt{2}}{4\pi\mu_0}\frac{r_i(-z+z_j)}{(2r_ir)^{3/2}\alpha_i^{(1)}}$
$\alpha_i^{(1)}$	$r_i^2 + r^2 + 2r_ir$
$\alpha_{i,j}^{(2)}$	$\sqrt{\frac{4r_ir}{\alpha_i^{(1)}+(z-z_j)^2}}$
$\alpha_i^{(3)}$	$\sqrt{\frac{\alpha_i^1+(z-z_j)^2}{4r_ir}}$
$\alpha_i^{(4)}$	$2r_ir^2 - r_i(r_i^2+r^2)$
$\alpha_i^{(5)}$	$\frac{\alpha_i^{(1)}+(z-z_j)^2}{\alpha_i^{(1)}}$
$\alpha_i^{(6)}$	$\sqrt{\frac{2(\alpha_i^{(1)}+(z-z_j)^2)}{4r_ir}}$

$$\mathbf{N}_{i,j} = \int_1^{-1}(1-u^2)\arctan\left[\frac{(r_i-ru)(z-z_j)}{\sqrt{r^2(u^2-1)\xi}}\right]du \tag{18}$$

$$\xi = \sqrt{r^2 + r_i^2 - 2rr_iu + (z-z_j)^2} \tag{19}$$

3.4. Axial Component $H_z(r,z)$

$$H_z(r,z) = \sum_{i=1}^{2}\sum_{j=1}^{2}(-1)^{(i+j)}\left(S_{i,j}^z + V_{i,j}^z\right) \tag{20}$$

Table 2. Parameters used for calculating the volume contribution of the radial component $H_r(r,\theta,z)$.

Parameters	
$\beta^{(1)}$	$a\sqrt{1-u^2}\sqrt{\frac{b-cu}{b+c}}+\frac{a\sqrt{c(1+u)}}{c\sqrt{1-u^2}}$
$\beta^{(2)}$	$\frac{a(a^2+b)\arcsin[u]}{c\sqrt{b+c}}\sqrt{b-cu}$
$\beta^{(3)}$	$(1+u)\sqrt{\frac{c(u-1)}{b-c}}$
$\beta^{(4)}$	$\arcsin\left[\sqrt{\frac{b-cu}{b+c}}\right]$
$\beta^{(5)}$	$\frac{b+c}{b-c}$
$\beta^{(6)}$	$\sqrt{1-u^2}\sqrt{\frac{c(1+u)}{b+c}}$
$\beta^{(7)}$	$\arcsin\left[\sqrt{\frac{1+u}{2}}\right]$
$\beta^{(8)}$	$\frac{2c}{b+c}$
$\beta^{(9)}$	$\frac{2c}{b+c-a^2}$
$\beta^{(10)}$	$-2\sqrt{1-u^2}\log[a+\sqrt{b-cu}]$
$\beta^{(11)}$	$-\frac{\sqrt{x}}{c}\log\left[\frac{4c^2\left(c+a^2u-bu+\sqrt{x}\sqrt{1-u^2}\right)}{x^{\frac{3}{2}}(a^2-b+cu)}\right]$
x	$-a^4+2a^2b-b^2+c$

with

$$S^z_{i,j} = \frac{2r_i}{(r-r_i)^2+(z-z_j)^2}\mathbf{K}^*\left[-\frac{4rr_i}{(r-r_i)^2+(z-z_j)^2}\right] \quad (21)$$

$$V^z_{i,j} = \int_0^{2\pi}\tanh^{-1}\left[\frac{\sqrt{r^2+r_i^2+(z-z_j)^2-2rr_i\cos(\theta_s)}}{r_i-r\cos(\theta_s)}\right] \quad (22)$$

The special functions used are defined as follows:

$$\mathbf{K}^*[m] = \mathbf{F}^*\left[\frac{\pi}{\mathbf{2}},\mathbf{m}\right] \quad (23)$$

$$\mathbf{F}^*[\phi,m] = \int_0^{\phi}\frac{1}{\sqrt{1-m\sin(\theta)^2}}d\theta \quad (24)$$

$$\Pi^*[n,\phi,m] = \int_0^{\phi}\frac{1}{(1-n\sin(\theta)^2)\sqrt{1-m\sin(\theta)^2}}d\theta \quad (25)$$

3.5. Comparison of the Magnetic Field Created by Ring Permanent Magnets Axially and Radially Magnetized

This section discusses the validity of the component switch theorem for ring permanent magnets axially and radially magnetized. For this purpose, we represent in Figs. 4 and 5 the magnetic field modulus $H(r, z)$ created by either a ring radially magnetized or a ring axially magnetized.

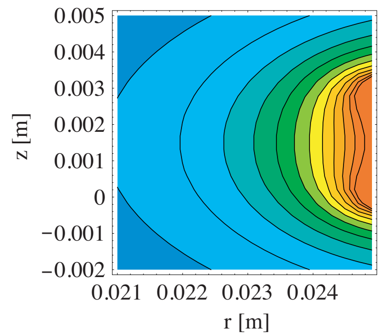


Figure 4. Modulus of the magnetic field created by a ring permanent magnet axially magnetized; we take $r = 0.024\,\text{m}$, $r_1 = 0.025\,\text{m}$, $r_2 = 0.028\,\text{m}$, $z_1 = 0\,\text{m}$, $z_2 = 0.003\,\text{m}$, $J = 1\,\text{T}$.

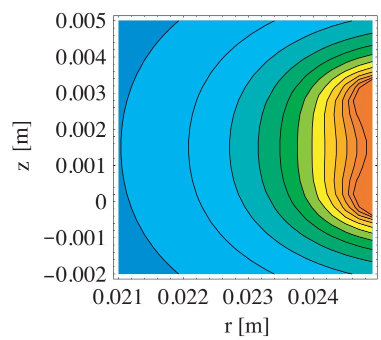
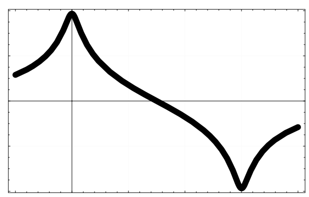


Figure 5. Modulus of the magnetic field created by a ring permanent magnet radially magnetized; we take $r = 0.024\,\text{m}$, $r_1 = 0.025\,\text{m}$, $r_2 = 0.028\,\text{m}$, $z_1 = 0\,\text{m}$, $z_2 = 0.003\,\text{m}$, $J = 1\,\text{T}$.

Figures 4 and 5 show that in the near-field, a ring permanent magnet radially magnetized is similar to a ring permanent magnet axially magnetized. The magnetic field becomes different when



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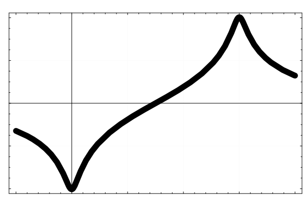

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3.7. Necessity of Taking into Account the Magnetic Pole Volume Density

This section explains why the magnetic pole volume density is necessary for calculating the magnetic field components created by a ring permanent magnet. This question is in fact crucial because the calculation of the force or the stiffness between ring permanent magnets is more complicated when the magnetic pole volume density is taken into account. To do so, we plot in Fig. 10 the relative difference of the radial field created by a ring permanent magnet versus the radial distance r with and without taking into account the magnetic pole volume density. In addition, we plot in Fig. 11 the relative difference of the axial field created by a ring permanent magnet versus the radial distance with and without the magnetic pole volume density.

We see that the farther the magnetic field is calculated from the magnets, the more the relative difference increases if the magnetic pole volume density is omitted. For a radial distance which equals 0.01 m from the magnets, we make an error of at least 25 per cent for the radial field and 20 per cent for the axial field is the magnetic pole volume density is omitted. Consequently, we deduct that we must take into account such a contribution for calculating the magnetic components in the far-field. For a radial distance which equals 0.001 m from the magnets, we make an error of at least 7 per cent for the radial field and 5 per cent for the axial field is the magnetic pole volume density is omitted. We can say that it is yet an important error for the near-field. Consequently, we must take into account the magnetic pole volume density for calculating the magnetic components in the near-field as well.

4. CONCLUSION

This paper has presented an improvement of the magnetic field calculation created by ring permanent magnets radially magnetized. The expressions obtained are based on real functions and the magnetic pole volume density is taken into account. We discuss the importance of taking into account such a contribution. Then, this paper discusses the validity of the component switch theorem for ring permanent magnets. This theorem can be used for describing the magnetic field created by infinite parallelepiped magnets (2D approach) whereas it cannot be used for modeling the magnetic field created by ring permanent magnets (3D approach). This result implies that an optimization of the ring dimensions with a 2D approach seems to be difficult. Eventually, we have compared, with the exact 3D approach, the magnetic field created by a ring permanent magnet radially magnetized with a ring permanent magnet axially magnetized. When their cross-section is a square, their magnetic field modulus is nearly the same in the near field whereas it is different in the far field. Such results imply that ring permanent magnets axially magnetized can be used in magnetic bearings in which the near-field is the most important parameter to optimize. The expressions given in this paper are available online [34].

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